Computer Simulation and Performance Evaluation

Homework 1

Due: 11/13/2000

1. Suppose that the number of arrivals in time [0,t] is given by a Poisson process with parameter. λ. Suppose we are told that there is exactly one arrival in [0,t]. Show that the arrival time of this customer is uniformly distributed in [0,t]; i.e., show that the probability that the customers has arrived before some time x, $0 < x < t$, is given by x/t. (10%)
2. Let X_1 and X_2 be two independent Poisson sources with parameters λ_1 and λ_2, respectively. Show that their sum $Y= X_1+X_2$ has a Poisson distribution with parameter $\lambda_1 + \lambda_2$. (10%)
3. Let X_1 and X_2 be independent exponentially distributed r.v.'s with parameters λ_1 and λ_2, respectively. Define a new r.v. Y as the minimum of X_1 and X_2. Find the CDF and pdf of Y. What kind of distribution does Y have? (10%)
4. Let X_1 , ..., X_n be n i.i.d.r.v.'s, each has an exponential distribution with parameter λ. How is $Y= X_1 + ...+ X_n$ distributed? (10%)
5. Consider the motion of a particle on the space {0, 1, 2, ...}. If the particle is in state $j \geqq 0$ at time n ($n \geqq 0$) then at time n+1 it will be either in state j+1 with probability $p_{j,j+1}$, in state j-1 with probability $p_{j,j-1}$, or in state j with probability $p_{j,j}$, where $p_{j,j+1}+ p_{j,j}+ p_{j,j-1}=1$ for all $j \geqq 0$, where by convention $p_{0,-1}=0$. Let X_n be the location of the particle at time n. (20%)
   (a) Say why (X_n, $n \geqq 0$) is a discrete-time discrete-state M.C. and write down its transition matrix P.
   (b) Given sufficient conditions under which this M.C. is irreducible and aperiodic.
   (c) We assume that X_n in {0,1,2} . Compute the limiting probabilities π_i, i=0, 1, 2 aas well as E[X] and var(X) when $p_{0,0}=0.25$, $p_{0,0}=0.25$, $p_{0,0}=0.75$, $p_{1,0}=0.1$, $p_{1,1}=0.3$, $p_{1,2}=0.6$, $p_{2,1}=0.4$, $p_{2,2}=0.6$, $p_{2,3}=0$.
6. Question 2.18 in textbook. (20%)
7. Question 2.19 in textbook. (20%)