· **Steady State Probability (stationary)**

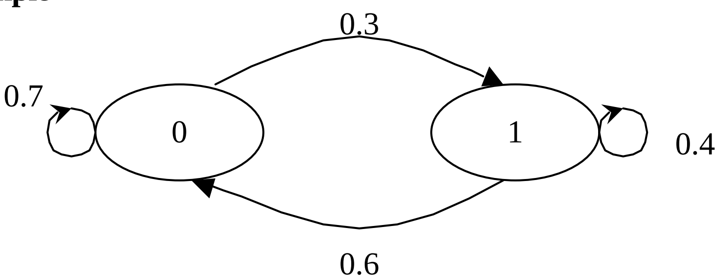
(1) $\lim_{n\to\infty} P_{ij}^{(n)} \approx \pi_j^{(n)}$

— prob(system is in state j after n steps)

(2) $\lim_{n\to\infty} \pi_j^{(n)} \approx \pi_j$

— prob(steady state probability for state j)

· **Example：**



$$P_{ij}^{(1)} = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$$

$$P_{ij}^{(4)} = \begin{bmatrix} 0.5749 & 0.4251 \\ 0.5668 & 0.4322 \end{bmatrix}$$

$$P_{ij}^{(20)} = \begin{bmatrix} 0.5714 & 0.4285 \\ 0.5714 & 0.4285 \end{bmatrix}$$

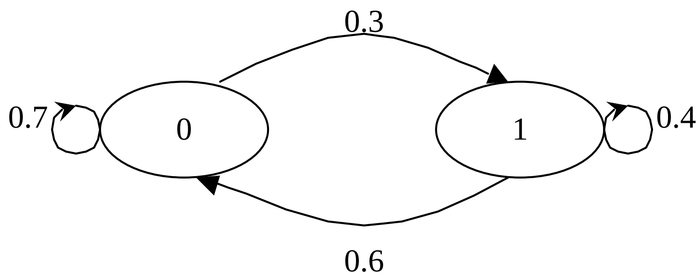
· **Thm：** For an ergodic Discrete time MC

(1) $\pi_j = \lim_{n\to\infty} \pi_j^{(n)}$ exist and

(2) $\{\pi_j\}$ are uniquely determined by the set of simulations linear equations

$$\begin{cases} \pi_j = \sum_{all\ i} \pi_i P_{ij} \quad , \nabla j \\ \sum_{all\ i} \pi_i = 1 \end{cases}$$

· **Example：**

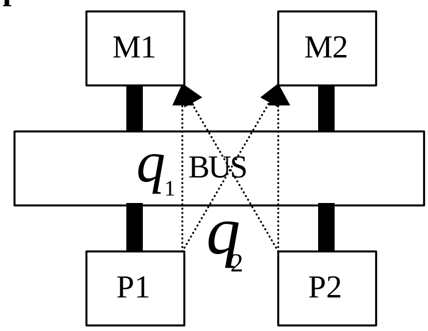


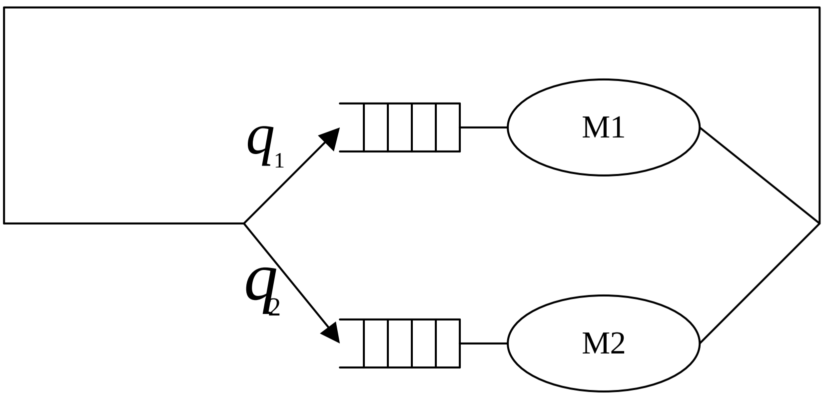
$$\begin{cases} p_0 = 0.7p_0 + 0.4p_1 \\ p_1 = 0.3p_0 + 0.6p_1 \end{cases}$$

又 $p_0 + p_1 = 1$

$$\Rightarrow \begin{cases} p_0 = \frac{4}{7} = 0.5714 \\ p_1 = \frac{3}{7} = 0.4285 \end{cases}$$

· **Example：**





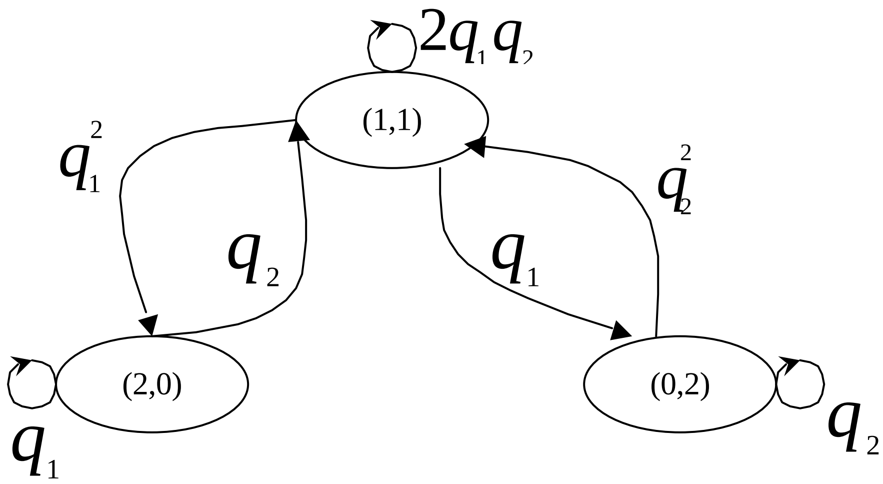
Process P_1， P_2 **向** memory M_1， M_2**要記憶體空間的** prob.**分別為**q_1

，q_2

Discrete time MC：state descriptor

(#reg. at M_1, #reg. at M_2)

(0 , 2)
(1 , 1)
(2 , 0)



$$\begin{cases} p_{11} = 2q_1q_2p_{11} + q_1p_{02} + q_2p_{20} \\ p_{02} = q_2p_{02} + q_2^2p_{11} \\ p_{20} = q_1p_{20} + q_1^2p_{11} \\ p_{11} + p_{02} + p_{20} = 1 \end{cases}$$

由以上 4 式，即可解出 p_{11}，p_{02} **和** p_{20}

$$\Rightarrow p_{11} = \frac{q_1q_2}{1-2q_1q_2}$$

$$E(through\ put) = 2\times p_{11} + 1\times p_{02} + 1\times p_{20} = \frac{1-q_1q_2}{1-2q_1q_2}$$

(**利用** $q_1 + q_2 = 1$ **的關係式，即可求出令** through put **最大的** q_1 **值**)

(**此例尚可利用一維的** state descriptor **來描述**)