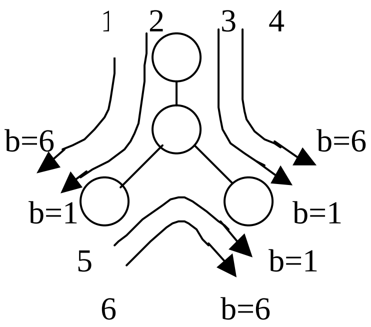
MDP for multirate loss network



Exact analysis

State space:

$$(X_1, X_2, X_3, X_4, X_5, X_6)$$

$$\sum_{k \in C_n} b_k X_k \le m_n$$

m_n: n-th Link's capacity

C_n: class that traverse Link n

Action

For each state

$$X = (X_1, \ldots, X_6)$$

$$A_X = \{a \mid (a_1, \ldots a_6)\}$$

$$a_i = \begin{cases} 0 \; block \;\; class \;\; i \\ 1 \; accept \;\; class \;\; i \end{cases}$$

Revenue

$$\sum_{i=1}^{6} (1 - q_i) r_i \lambda_i$$

<u>**Decomposed MDP**</u>

-look at a single Link only

-Assume initial policy is CS(complete share) policy

-Form a MDP at each link

-classify calls according to bandwidth requirement

e.g. $X=(X_1,X_2)$ $b_1=6$ $b_2=1$

-modified link reward

・ for original class i calls, i=1,…,6

modified link reward= $\dfrac{r_i}{|R_i|}$ $|R_i|$: length of routing path

・ for aggregated class $r_i' = \sum_j \dfrac{\lambda}{\sum_j \lambda_j} \times \dfrac{r_i}{|R_i|}$

$$\begin{cases} \lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 2, \lambda_4 = 2 \\ \gamma_1 = 6, \gamma_2 = 1, \gamma_3 = 12, \gamma_4 = 2 \end{cases}$$

$$mr_1 = \frac{6}{2} = 3 \qquad mr_2 = \frac{1}{2} \qquad mr_3 = 6 \qquad mr_4 = 1$$

$$\gamma_1^1 = \frac{1}{1+2} 3 + \frac{2}{1+2} 6$$

$$\gamma_2^1 = \frac{1}{1+2} \times \frac{1}{2} + \frac{2}{1+2} \times 1$$

state descriptor:

(X_1, X_2)

Action:

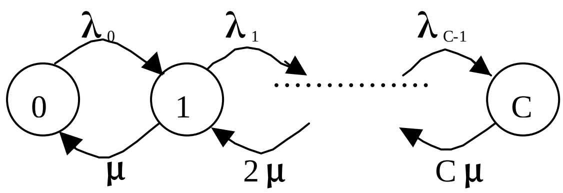
(a_1, a_2) , $a_i = \begin{cases} 0 \\ 1 \end{cases}$

Reward (loss):

$$\sum (1 - a_i) \cdot \gamma_i ' \cdot \lambda_i '$$

<u>Decomposed MDP :</u> Link indep. + Approximation

Reduce to one-dimension CTMC state descriptor: (# of busy circuits)



$$\lambda_i = \frac{\ \ 2}{\ \ 2} + i(1 - \frac{t}{\ \ 2})$$

$$_i = \frac{\lambda_i}{\ \ _i}$$

$$= \sum_{i=1}^{k} b_i \ \ _i \qquad\qquad ^2 = \sum_{i=1}^{k} b_i^2 \ \ _i$$

$$g = 0 + \overline{\lambda_0}(V_1 - V_0)$$

$$g = 0 + \overline{\lambda_1}(V_2 - V_1) - \quad (V_1 - V_0)$$

$$\ldots\ldots$$

$$g = 0 + \overline{\lambda_{C-1}}(V_C - V_{C-1}) - (C-1) \quad (V_{C-1} - V_{C-2})$$

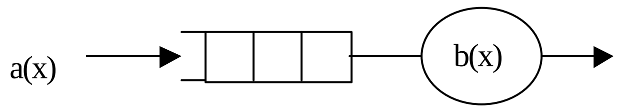
$$g = \sum_{i=1}^{k} \gamma_i \lambda_i - C \quad (V_C - V_{C-1})$$

$$\Rightarrow \begin{cases} W_i = V_{i+1} - V_i = \dfrac{g}{\overline{\lambda_i} E(\bar{\lambda}, i)} = (V_{i+bk} - V_{i+bk-1}) + \ldots = W_{i+bk-1} + \ldots + W_i \\ g = \sum_{i=1}^{k} \gamma_i \lambda_i - C W_{C-1} \\ W_{C-1} = \dfrac{\sum \gamma_i \lambda_i E(\bar{\lambda}, C)}{\overline{\lambda_{C-1}} E(\bar{\lambda}, C-1)} \end{cases}$$

Policy improvement:

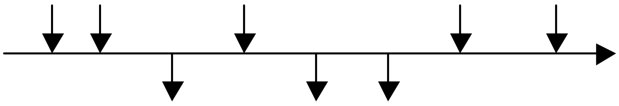
State i: accept class k if $\gamma_k > (V_{i+bk} - V_i) / \quad _k$

M/G/1 Queue



- service time are no longer exponentially distribution
  b(x): service time pdf
  a(x): $\lambda e^{-\lambda x}$ Poisson process
- model this system as a CTMC need two state variable (# in queue, remaining service time of the customer in service)

solution: Embedded Markov



state descriptor : (# in queues)
track the system state only after m states of departure

P_K^D =Prob(departure sees k customers left behind)

P_K^A = Prob(arriving customer sees k customers in queue)

P_K^R = Prob(random observer sees k customers in queue)

$P_K^R = P_K^A = P_K^D$

PASTA

Define:

$a_n(T)$: # of occurances of arrivals that changes state to n from (n-1)

$d_n(T)$: # of occurances of departure that changes state to n form (n+1)

Lemma 1 : $|a_n(T) - d_n(T)| \leq 1$

Lemma 2 : $d(T) = a(T) + n(0) - n(T)$

$$P_K^D = \lim_{T\to\infty} \frac{d_k(T)}{d(T)}$$

$$= \lim_{T\to\infty} \frac{a_k(T) + d_k(T) - a_k(T)}{a(T) + n(0) - n(T)}$$

$$= \lim_{T\to\infty} \frac{a_k(T)}{a(T)} = p_K^A$$

Constrain

single server → single departure

Poisson arrival → PASTA