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$E[T] = \frac{1}{\mu} + \frac{\lambda}{2(1-\rho)\mu^2}(1+C_b{}^2)$

$\lambda = 0.5$ job / sec

$\mu = 1$ job / sec

case 1：constant service time

$C_b = 0$

$E[T] = 1 + \frac{0.5}{2(0.5)} \times 1 = 1.5$ sec

case 2：exponential service time

$C_b = 1$

$E[T] = 1 + \frac{0.5}{2(0.5)} \times 2 = 2$ sec

case 3：hyper exponential service time

$C_b = 3$

$E[T] = 1 + \frac{0.5}{2(0.5)} \times 4 = 3$ sec

M / G / 1

Method2 for E[T]

Define : n_i：number of customers in system right after the ith departure

a_i：number of arrivals during the ith service

$n_{i+1} = n_{i-1} + a_{i+1}$, if $n_i > 0$

a_{i+1} , if $n_i = 0$

Define $u(n_i) = 1$, $n_i > 0$

0 , $n_i = 0$

$n_{i+1} = n_i - u(n_i) + a_{i+1}$

$$E[a_{i+1}] = \int_0^\infty E[a_{i+1} | servicetime = y] G(y) dy$$

$$= \int_0^\infty l y G(y) dy$$

$$= l \int_0^\infty y G(y) dy$$

$$= l\bar{x}$$

$$= P$$

Method 2 for E[T]

$E[n_{i+1}] = E[n_i] - E[u(n_i)] + E[a_{i+1}]$

1) $E[u(n_i)] = E[a_{i+1}] = \rho$

2) $E[n_{i+1}^2] = E[n_i^2] + E[(u(n_i))^2] + E[a_{i+1}^2] - 2\, E[n_i\, u(n_i)] + 2\, E[a_{i+1}\, n_i]$

$- 2\, E[u(n_i) a_{i+1}]$

$E[(u(n_i))^2] = E[u(n_i)] = \rho$

$E[n_i\, u(n_i)] = E[n_i]$

$E[u(n_i) a_{i+1}] = E[u(n_i)]\, E[a_{i+1}]$

$E[a_{i+1}\, n_i] = E[a_{i+1}]\, E[n_i]$

$E[n_{i+1}^2] = E[n_i^2] + E[u(n_i)] + E[a_{i+1}^2] - 2\, E[n_i] + 2\rho\, E[n_i] - 2\rho\, E[u(n_i)]$

$= E[n_i^2] + \rho + E[a_{i+1}^2] - 2\, E[n_i] + 2\rho\, E[n_i] - 2\rho^2$

$\because\ i \to \infty$

$E[n_{i+1}] = E[n_i] = E[n]$

$E[n^2] = E[n^2] + \rho + E[a^2] - 2E[n] + 2\rho E[n] - 2\rho^2$

$2(1-\rho)E[n] = \rho + E[a^2] - 2\rho^2$

$$E[n] = \rho + \frac{E[a^2] - r}{2(1-r)}$$

$$E[a^2] = \int_0^\infty E[a^2 \mid servicetim\ e = y] G(y) dy$$

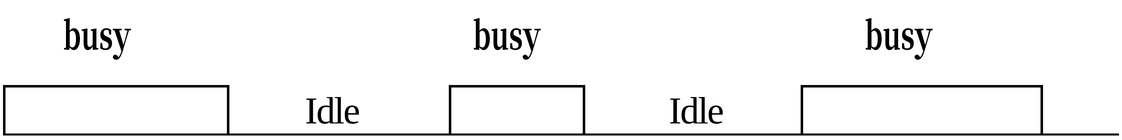
$$= \int_0^\infty [(l y)^2 + l y] G(y) dy$$

$$= l^2 \int_0^\infty y^2 G(y) dy + l \int_0^\infty y G(y) dy$$

$$= \lambda^2 E[x^2] + \rho$$

$$E[n] = \rho + \frac{l^2 E[x^2]}{2(1-r)}$$

Busy period of M / G / 1



I_n : idle time during the nth regeneration period

B_n : busy time during the nth regeneration period

P_o : prob that the server is idle

$$P_o = \lim_{n\to\infty} \frac{I_1 + I_2 + \ldots + I_n}{(I_1 + I_2 + \ldots + I_n) + (B_1 + B_2 + \ldots + B_n)} = \frac{E[I]}{E[I] + E[B]}$$

$P_o = 1 - \rho$

$$E[I] = \frac{1}{l}$$

$$E[B] = \frac{\bar{x}}{1-r}$$